The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING 2008-2009

SOLID MECHANICS 3

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer THREE questions

Only silent, self contained calculators with a Single-Line Display, or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL: None

INFORMATION FOR INVIGILATORS: None

The following formulae may be used without proof:

For beams in bending under elastic behaviour:

$$EI\frac{d^2y}{dx^2} = -M$$

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

For shafts in torsion under elastic behaviour:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{\ell}$$

For thick pressurised cylinders:

$$\sigma_r = A - \frac{B}{r^2} \quad \text{and} \quad \sigma_\theta = A + \frac{B}{r^2}$$

$$\varepsilon_r = \frac{du}{dr} \quad \text{and} \quad \varepsilon_\theta = \frac{u}{r}$$

1. (a) Show that the critical buckling load, P_c for a hinged-hinged ideal strut (ie, initially straight with purely axial loading) under compressive axial load is given by the following formula

$$Pc = \frac{\pi^2 EI}{\ell^2}$$

Where E is the Young's modulus, I is the minimum second moment of area of the cross section and ℓ is the length between the hinges. [40 marks]

(b) i) The maximum deflection, $\hat{y}$, at the midspan of an eccentrically-loaded strut of length ℓ is given by the expression

$$\hat{y} = e[\sec\left(\frac{\alpha l}{2}\right) - 1]$$

Where e is the eccentricity of the compressive axial load which acts parallel to the axis of the strut and $\alpha = \sqrt{P/EI}$ where P is the applied load and E,I are as in (a). If the maximum deflection of such a 2 m long, eccentrically – loaded steel strut, of square cross-section 36 mm x 36 mm, is 20 mm under an axial compressive load of 20kN, determine the eccentricity of the load. Assume E=200 GPa for steel. [30 marks]

ii) Hence determine the maximum compressive and tensile stresses in the strut. [30 marks]

2. A pair of mild steel cylinders (E=207 GPa) of equal length have the following dimensions:

- 30 mm bore and 70.10 mm outside diameter
- 70 mm bore and 110 mm outside diameter

ie. there is a diametral interference of 0.1 mm. The larger cylinder is heated up and placed around the smaller cylinder and allowed to shrink onto it.

(a) Calculate the resulting interference pressure between the two cylinders. [40 marks]

(b) Hence obtain expressions for the variations of hoop and radial stresses with radius and sketch these distributions indicating the hoop stress values at the inside radius of each cylinder. [60 marks]

3. (a) Explain, with the aid of diagrams, the elastic-perfectly-plastic, the isotropic hardening and the kinematic hardening models of elastic plastic material behaviour. In each case, sketch and label the typical evolution of the yield surfaces in the three-dimensional principal stress space. [30 marks]

(c) A solid circular shaft, of diameter 80 mm and length 2 m, is subjected to a torque, T. The shaft is made from an elastic-perfectly-plastic material with $\tau_y = 100 N/mm^2$ and $G = 70\ GN/m^2$, where τ_y is the shear yield stress and G is the shear modulus.

i) Determine the magnitude of the torque required to cause yielding to occur to a radius of 25 mm (and greater) and the angle of twist in the loaded state. [35 marks]

ii) Determine the residual stress distribution and angle of twist when the shaft is subsequently unloaded. [35 marks]

4. (a) A steel cylinder has an inside diameter of 50 mm and a wall thickness of 1mm. If the cylinder has closed ends and the material has a yield stress of 250MPa, determine the pressure required to cause yielding according to both the Tresca and von Mises yield criteria, using thin walled pressure vessel theory. [50 marks]

(b) If a torque of 0.25kNm is superimposed on the cylinder of (a) determine the new pressure required to cause yielding using the Tresca yield criterion only. [50 marks]

5. (a) Describe the total life approach and the damage tolerant approach to fatigue life prediction in engineering components, using equations and diagrams to show the key aspects of each approach. [40 marks]

(b) i) Explain, using sketches and equations, the principle of virtual work. [20 marks]

ii) Describe, using diagrams and equations where necessary, the key stages involved in deriving the stiffness matrix for an element used for finite element analysis. [40 marks]

End